ISSN 1064-5624, Doklady Mathematics, 2013, Vol. 88, No. 3, pp. 737–740. © Pleiades Publishing, Ltd., 2013.
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MATHEMATICS

On Graphs Whose Local Subgraphs Are Strongly Regular with Parameters (99, 14, 1, 2)

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Received June 25, 2013

DOI: 10.1134/S106456241306032X

We consider undirected graphs without loops or multiple edges. If a and b are vertices in a graph Γ, then $d(a, b)$ denotes the distance between a and b, and $\Gamma_i(a)$ denotes the subgraph of Γ induced by the set of vertices of Γ that are a distance of i away from a. The subgraph $\Gamma_1(a)$ is called the neighborhood of a and is denoted by $[a]$.

Γ is called a regular graph of degree k if $[a]$ contains precisely k vertices for any vertex a in Γ. Γ is called an amply regular graph with parameters (v, k, λ, μ) if Γ is a regular graph of degree k on v vertices, and each edge of Γ lies in λ triangles, and the subgraph $[a] \cap [b]$ contains μ vertices in the case $d(a, b) = 2$. An amply regular graph of diameter 2 is called a strongly regular graph.

A graph Γ of diameter d is said to be antipodal if the relation of coincidence or being a distance of d apart on its vertex set is an equivalence relation. The antipodal quotient Γ' is a graph whose vertices are the antipodal classes of Γ and two antipodal classes are adjacent if a vertex of one class is adjacent to a vertex of the other class. An antipodal graph Γ is called an r-covering (of its antipodal quotient) if each of its antipodal classes contains precisely r vertices.

Let $K_{m_1, \ldots, m_n}$ denote a complete n-partite graph with parts of orders $m_1, m_2, \ldots, m_n$. If $m_1 = m_2 = \ldots = m_n = m$, then this graph is denoted by $K_{n \times m}$.

If vertices u and w are separated by a distance of i in Γ, then $b_i(u, w)$ ($c_i(u, w)$) denotes the number of vertices in the intersection of $\Gamma_{i+1}(u)$ ($\Gamma_{i-1}(u)$) with $[w]$. A graph of diameter d is called a distance-regular graph with an intersection array $\{b_0, \ldots, b_{d-1}; c_1, \ldots, c_d\}$ if $b_i(u, w)$ and $c_i(u, w)$ are independent of the choice of the vertices u and w separated by the distance i. Let $a_i = k - b_i - c_i$ and $k_i = |\Gamma_i(u)|$ (k_i is independent of the choice of the vertex u).

In [1] a program was proposed for the study of distance-regular graphs whose local subgraphs (neighborhoods of vertices) are strongly regular with an eigenvalue of 3. The problem in [1] was reduced to the case where neighborhoods of vertices belong to a finite set of graphs.

Graphs whose local subgraphs are strongly regular with $\lambda = 1$ are of special interest. The well-known strongly regular graph with $\lambda = 1$ is the point graph of the generalized quadrangle $GQ(2, t)$ for $t = 1, 2, 4$ or a graph with parameters (81, 20, 1, 6), (243, 22, 1, 2), or (729, 112, 1, 20). The amply regular graphs whose local subgraphs are the point graphs of generalized quadrangles $GQ(2, t)$ and a graph with parameters (81, 20, 1, 6) were classified in [2] and [3], respectively.

It is not known whether there exist strongly regular graphs with parameters (99, 14, 1, 2) and (115, 18, 1, 3) and with an eigenvalue of 3.

In this paper, we classify the distance-regular graphs whose local subgraphs are isomorphic to a strongly regular graph with parameters (99, 14, 1, 2).

Theorem. *Let Γ be a distance-regular graph whose local subgraphs are strongly regular with parameters (99, 14, 1, 2). Then one of the following assertions holds:*

(1) *Γ is an antipodal graph with the intersection array $\{99, 84, 1; 1, 14, 99\}$ or $\{99, 84, 1; 1, 12, 99\}$ and wit the spectrum $99^1, \sqrt{99}^{300}, -1^{99}, -\sqrt{99}^{300}$ or $99^1, 11^{315}, -1^{99}, -9^{385}$, respectively.*

(2) *Γ is a primitive graph with the intersection array $\{99, 84, 30; 1, 6, 54\}$ and the spectrum $99^1, 27^{141}, 5^{1080}, -9^{1034}$.*

Let us present some auxiliary results.

Lemma 1. *Let Γ be a strongly regular graph with parameters (v, k, λ, μ). Then either $k = 2\mu$ and $\lambda = \mu - 1$ (so-called half case) or the nonprincipal eigenvalues $n - m$ and $-m$ of Γ are integers, where $n^2 = (\lambda - \mu)^2 + 4(k - \mu)$, $n - \lambda + \mu = 2m$, and the multiplicity of $n - m$ is $\dfrac{k(m-1)(k+m)}{\mu n}$. Moreover, if m is an integer larger than 1, then $m - 1$ divides $k - \lambda - 1$ and*

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$$\mu = \lambda + 2 + (m-1) - \frac{k-\lambda-1}{m-1},$$

$$n = m - 1 + \frac{k-\lambda-1}{m-1}.$$

Proof. This is Lemma 3.1 from [4].

Lemma 2. *Let* Γ *be a strongly regular graph with parameters* (v, k, λ, μ), *and let* Δ *be an induced subgraph on N vertices of degrees* $d_1, \ldots, d_N$ *with M edges. Then*

$$(v-N)-(kN-2M)$$
$$+\left(\lambda M+\mu\left(\binom{N}{2}-M\right)-\sum_{i=1}^{N}\binom{d_i}{2}\right)$$
$$= x_0+\sum_{i=3}^{N}\binom{i-1}{2}x_i,$$

where $x_i = x_i(\Delta)$.

Lemma 3. *Let* Γ *be a strongly regular graph with parameters* $(99, 14, 1, 2)$ *and eigenvalues* 3 *and* -4, Δ *be a second-degree regular subgraph of* Γ *on n vertices,* X_i *be the set of vertices of* $\Gamma - \Delta$ *that are adjacent to precisely i vertices from* Δ, *and* $x_i = |X_i|$. *Then the following assertions hold:*

(1) $\sum x_i = 99 - n$, $\sum i x_i = 12n$, $\sum\binom{i}{2}x_i = n + 2\left(\frac{n(n-1)}{2} - n\right) - n = n^2 - 3n$, and $x_0 + \sum\binom{i-1}{2}x_i = 99 + n^2 - 16n$.

(2) $n \le 33$.

(3) *If* $n = 4$, *then* $x_0 = 51$; *if* $n = 6$, *then* $x_0 \le 39$; *if* $n = 7$, *then* $x_0 \le 36$; *and if* $n = 9$, *then* $x_0 \le 36$.

Proof. By Lemma 2, we have $\sum x_i = 99 - n$, $\sum i x_i = 12n$, and $\sum\binom{i}{2}x_i = n + 2\left(\frac{n(n-1)}{2} - n\right) - n = n^2 - 3n$. Therefore, $x_0 + \sum\binom{i-1}{2}x_i = 99 + n^2 - 16n$. Assertion (1) is proved.

In view of [5], we have $-4 \le 2 - \frac{12n}{99-n} \le 3$. Therefore, $n \le 33$. Moreover, if $n = 33$, then each vertex of $\Gamma - \Delta$ is adjacent to precisely $\frac{12 \cdot 33}{99-33} = 6$ vertices in Δ.

If $n = 4$, then $x_2 = 4$, $x_0 = 51$, and $x_1 = 40$. If $n = 5$, then $x_0 \le 99 - 55 = 44$. If $n = 6$, then $x_0 \le 99 - 60 = 39$. If $n = 7$, then $x_0 \le 99 - 63 = 36$. If $n = 9$, then $x_0 \le 99 - 63 = 36$.

Lemma 4. *Let* Γ *be a strongly regular graph with parameters* $(99, 14, 1, 2)$, *and let X and Y be subsets of vertices of* Γ *such that there are no edges between X and Y. Then* $|X| \cdot |Y| \le \frac{(99-|X|)(99-|Y|)\cdot 7^2}{29^2}$ *and, if* $|X| = |Y|$, *then* $|X| \le 19$.

Proof. Since there are no edges between X and Y, Proposition 4.6.1 in [6] implies that $|X| \cdot |Y| \le \frac{(v-|X|)(v-|Y|)(\theta_2-\theta_1)^2}{(2k-\theta_2-\theta_1)^2}$, where $\theta_1 = 3$ and $\theta_2 = -4$ are nonprincipal eigenvalues of Γ. From this, $|X| \cdot |Y| \le \frac{(99-|X|)(99-|Y|)\cdot 7^2}{29^2}$.

If $|X| = |Y|$, we have $29|X| \le 7(99 - |X|)$ and $|X| \le 19$.

Lemma 5. *Let* Γ *be a distance-regular graph of diameter* $d \ge 3$ *whose local subgraphs are strongly regular with parameters* $(99, 14, 1, 2)$, *and let* $\theta_0 = k > \theta_1 > \ldots > \theta_d$ *be the eigenvalues of* Γ. *Then* $\theta_1 \le 27$ *and* $\theta_d \ge -20$.

Proof. By Terwilliger's theorem [7, Theorem 4.4.3], $-4 \ge b^- = -1 - \frac{b_1}{\theta_1+1}$, $3 \le b^+ = -1 - \frac{b_1}{\theta_d+1}$. Therefore, $\theta_1 \le 27$ and $\theta_d \ge -20$.

In what follows, let Γ be a connected amply regular graph of diameter d whose local subgraphs are strongly regular with parameters $(99, 14, 1, 2)$. Fix a vertex u in Γ and set $k_i = |\Gamma_i(u)|$.

Lemma 6. *The following assertions hold*:

(1) *The diameter of* Γ *is larger than* 2 *and* $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18, 21, 22, 27, 28\}$.

(2) *If the diameter of* Γ *is larger than* 3, *then* $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18\}$.

(3) *If the diameter of* Γ *is larger than* 4, *then* $\mu < 18$.

Proof. By assumption, $k = 99$ and $\lambda = 14$. If the diameter of Γ is 2, then, by Lemma 1, the number $(\lambda - \mu)^2 + 4(k - \mu)$ is the square of a positive integer n. Therefore, $(\mu - 16)^2 + 336 = n^2$ and $(\mu, n) \in \{(8, 20), (11, 19), (21, 19), (24, 20), (33, 25)\}$. Consequently, Γ has the eigenvalues $13, -7$; $11, -8$; $6, -13$; $5, -15$; or $3, -22$. In any case, the multiplicities of the eigenvalues are not integers.

Let the diameter of Γ be larger than 2. By Lemma 3, $\mu < 33$. Since μ is a divisor of $99 \cdot 84$, we have $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18, 21, 22, 27, 28\}$. Assertion (2) is proved.

Let the diameter of Γ be larger than 3, and let u, w, x, y, z be a geodesic 4-path in Γ. Then there are no edges between $[u] \cap [x]$ and $[x] \cap [z]$ in the graph $|x|$ and, by Lemma 4, we have $\mu \le 18$. From this, $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18\}$.

Let the diameter of Γ be larger than 4. Then $\frac{3\mu}{2} \le c_3 \le b_2$ and $\mu \ne 18$.

Lemma 7. *The parameter* μ *is at most* 18.

Proof. Assume that $\mu > 18$. By Lemma 6, the diameter of Γ is 3 and $\mu \in \{21, 22, 27, 28\}$.

If $\mu = 28$, then $k_2 = 99 \cdot 3$ and, by Lemma 4, we have $29^2 \cdot 4b_2 \le 7 \cdot 71(99 - b_2)$. Therefore, $b_2 \le 12$, $c_3 \ge 28 - b_2 + 16$ and $c_3 \in \{33, 36, 44, 45, 54, 55, 63, 66, 72, 77, 81, 88, 90, 99\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 27$. Then $k_2 = 11 \cdot 28$ and, by Lemma 4, we have $b_2 \le 12$. Furthermore, b_2 is divided by 9 and $c_3 \in \{36, 42, 44, 54, 56, 63, 66, 72, 88, 99\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 22$. Then $k_2 = 9 \cdot 42$ and, by Lemma 4, we have $b_2 \le 15$. Furthermore, b_2 is even and b_1b_2 is divided by 11, a contradiction.

Let $\mu = 21$. Then $k_2 = 9 \cdot 44$ and, by Lemma 4, we have $b_2 \le 17$. If $c_3 \in 66$, then $\theta_1 > 27$, a contradiction. Hence, $c_3 \in \{66, 68, 72, 77, 81, 84, 88, 90, 96, 99\}$. If $b_2 = 12$ and $c_3 = 88$, the graph has the integer eigenvalues 15, -1, -22. In any case, there are no admissible intersection arrays. The lemma is proved.

In Lemmas 8 and 9, the diameter of Γ is assumed to be 3.

Lemma 8. *If* $9 < \mu \le 18$, *then* Γ *has the intersection array* $\{99, 84, 1; 1, 14, 99\}$ *or* $\{99, 84, 1; 1, 12, 99\}$.

Proof. Assume that $9 < \mu \le 18$.

Let $\mu = 18$. Then $k_2 = 11 \cdot 42$ and, by Lemma 4, $b_2 \le 20$ and b_2 is divided by 3. If $c_3 < 66$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{66, 70, 72, 84, 99\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 14$. Then $k_2 = 11 \cdot 54$ and, by Lemma 4, we have $b_2 \le 25$. If $c_3 < 66$, then $\theta_1 > 27$, a contradiction. If $c_3 > 66$, then Γ has the intersection array $\{99, 84, 1; 1, 14, 99\}$ and the spectrum 99^1, $\sqrt{99}^{300}$, -1^{99}, $-\sqrt{99}^{300}$.

Let $\mu = 12$. Then $k_2 = 11 \cdot 63$ and, by Lemma 4, we have $b_2 \le 29$. Furthermore, a_2 is even, while b_2 and c_3 are odd. If $c_3 < 63$, then $\theta_1 > 27$, a contradiction. If $c_3 > 66$, then Γ has the intersection array $\{99, 84, 1; 1, 12, 99\}$ and the spectrum 99^1, 11^{315}, -1^{99}, -9^{385}.

Let $\mu = 11$. Then $k_2 = 27 \cdot 28$ and, by Lemma 4, $b_2 \le 31$ and b_2 is divided by 11. If $c_3 < 60$, then $\theta_1 < 27$, a contradiction. Therefore, $c_3 \in \{63, 72, 84\}$. In any case, there are no admissible intersection arrays.

Lemma 9. *If* $4 \le \mu \le 9$, *then* Γ *has the intersection array* $\{99, 84, 30; 1, 6, 54\}$.

Proof. Assume that $4 \le \mu \le 9$.

Let $\mu = 9$. Then $k_2 = 11 \cdot 84$ and, by Lemma 3, $b_2 \le 36$ and b_2 is divided by 3. If $c_3 < 54$, then $\theta_1 > 27$, a contradiction. If $c_3 \ge 54$, there are no admissible intersection arrays.

Let $\mu = 7$. Then $k_2 = 99 \cdot 12$ and, by Lemma 3, we have $b_2 \le 36$. If $c_3 < 52$, then $\theta_1 > 27$, a contradiction. If $c_3 \ge 53$, there are no admissible intersection arrays.

Let $\mu = 6$. Then $k_2 = 99 \cdot 14$ and, by Lemma 3, we obtain $b_2 \le 39$. If $c_3 = 54$, then Γ has the intersection array $\{99, 84, 30; 1, 6, 54\}$. In the other cases, there are no admissible intersection arrays.

Let $\mu = 4$. Then $k_2 = 99 \cdot 21$ and, by Lemma 3, we obtain $b_2 \le 51$. Furthermore, a_2 is even, while b_2 and c_3 are odd. If $c_3 < 38$, then $\theta_1 > 27$, a contradiction. In any case, there are no admissible intersection arrays.

Lemma 10. *If* $d(\Gamma) = 4$, *then there are no admissible intersection arrays.*

Proof. Let $\mu = 18$. Then $k_2 = 11 \cdot 42$. By Lemma 4, $b_2 \le 20$ and b_2 is divided by 3. Therefore, $b_2 = 18$. If $c_3 \le 66$, then $\theta_1 > 27$, a contradiction. Thus, $c_3 \in \{77, 84\}$, a contradiction to $\theta_4 < -20$.

Let $\mu = 14$. Then $k_2 = 11 \cdot 54$. By Lemma 4, we have $b_2 \le 25$. If $c_3 < 66$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{66, 69, 72, 75, 77, 81, 84\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 12$. Then $k_2 = 99 \cdot 7$ and, by Lemma 4, we have $12 \le b_2 \le 29$. Furthermore, a_2 is even. Therefore, b_2 and c_3 are odd. If $c_3 \le 64$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{69, 75, 77, 81\}$. If $c_3 \le 77$, then $\theta_1 > 27$. Thus, $c_3 = 77$ and $c_4 = 84$. In this case, there are no admissible intersection arrays.

Let $\mu = 11$. Then $k_2 = 9 \cdot 84$ and, by Lemma 4, $11 \le b_2 \le 31$ and b_2 is divided by 11. If $c_3 \le 60$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{66, 77, 84\}$. In any case, there are no admissible intersection arrays.

The cases $\mu = 4, 6, 7, 9$ are treated in a similar fashion. The lemma is proved.

Throughout the rest of this paper, we assume that $d = d(\Gamma) \ge 5$.

Lemma 11. *The following assertions hold:*

(1) *If* $\mu = 14$, *then* $d \le 4$, *while, if* $\mu = 11, 12$, *then* $d \le 5$.

(2) *If* $\mu = 9$, *then* $d \le 6$, *while, if* $\mu = 4, 6, 7$, *then* $d \le 7$.

Proof. By Theorem 5.2.1 in [7], we have $c_3 - b_3 \ge c_2 - b_2 + 16, \ldots, c_i - b_i \ge c_{i-1} - b_{i-1} + 16$. Summing up the inequalities termwise yields $c_i - b_i \ge c_2 - b_2 + (i - 2) \cdot 16$.

If $\mu = 14$, then $k_2 = 99 \cdot 6$ and, by Lemma 4, we have $b_2 \le 25$ and $c_3 - b_3 \ge 14 - b_2 + 16$. Therefore, $d \le 5$. In the case $d = 5$, we obtain $b_3 \le c_3 - 5 \le b_2 - 5$. If $c_3 = 19$, then $b_2 = 19$ and $b_3 \le 8$, a contradiction. Therefore, $c_3 \ge 20$ and, if $b_2 \ne 21$, then $b_3 = 14$, which contradicts the fact that c_3 divides $99 \cdot 84$. Therefore, $b_2 = 21$ and $b_3 \le c_3 - 9$, a contradiction to the fact that c_3 divides $99 \cdot 126$.

If $\mu = 12$, then $k_2 = 99 \cdot 7$ and a_2 is even, while b_2 and c_3 are odd. Furthermore, by Lemma 4, we have $b_2 \le 29$ and $c_3 - b_3 \ge 12 - b_2 + 16$. Therefore, $d \le 6$. If $d = 6$, then $b_2 = 29$ and $c_3 - b_3 = -1$. Therefore, b_3 is divided by 12 and either $b_3 = 24$ and $c_3 = 23$ or $b_3 = 12$ and $c_3 = 11$. In any case, we have a contradiction.

If $\mu = 11$, then, by Lemma 4, $b_2 \le 31$ and b_2 is divided by 11. Therefore, $b_2 \le 22$. Furthermore, $c_3 - b_3 \ge 11 - b_2 + 16$ and $d \le 5$.

If $\mu = 9$, then $k_2 = 11 \cdot 84$ and b_2 is divided by 3. By Lemma 3, we have $b_2 \le 36$. Therefore, $c_4 - b_4 \ge 9 - b_2 + 32$ and $d \le 7$. Let $d = 7$. Then $b_4 \le c_4 - 5$ and, if $b_4 \ge 17$, then, by Lemma 4, we have $c_4 \le 21$, a contradiction. Therefore, $c_3 \le b_4 \le 16$. If $c_3 = 14$, then, by Lemma 4, $c_4 \le b_3 \le 25$. Since c_4 is divided by 9, we have $c_4 = 18$

and $b_4 < 13$, a contradiction. If $c_3 = 16$, then c_4 is again divided by 9 and, by Lemma 4, we have $c_4 \le 18$, a contradiction. Therefore, $c_3 = 15$; b_2 is divided by 15; c_4 is divided by 3; and, by Lemma 4, we have $c_4 \le b_3 \le 24$. Now $b_4 \le c_4 - 11$. If $b_3 = 18$, then $c_4 \le 18$ and $b_4 \le 13$, a contradiction. From this, b_3 is not divided by 9, b_4 is divided by 3, and $b_4 = 15$. Therefore, $c_4 \le 26$, a contradiction.

If $\mu = 7$, then, by Lemma 3, we have $b_2 \le 36$. Therefore, $c_4 - b_4 \ge 7 - b_2 + 32$ and $d \le 7$.

If $\mu = 6$, then $k_2 = 99 \cdot 14$ and, by Lemma 3, we have $b_2 \le 39$. Therefore, $c_4 - b_4 \ge 6 - b_2 + 32$ and $d \le 8$. Let $d = 8$. Then $c_4 \le b_4 \le c_4 + 1$. If $c_4 = b_4$, then, by Lemma 4, we have $b_4 \le 19$, a contradiction to $0 = c_4 - b_4 \ge c_3 - b_3 + 16$. Therefore, $c_4 + 1 = b_4$ and, by Lemma 4, we have $b_4 \le 20$, a contradiction to $-1 = c_4 - b_4 \ge c_3 - b_3 + 16$.

If $\mu = 4$, then $k_2 = 99 \cdot 21$, a_2 is even, b_2 and c_3 are odd, and b_3 and c_4 are divided by 4. Furthermore, by Lemma 3, we have $b_2 \le 51$. Therefore, $c_5 - b_5 \ge 4 - b_2 + 48 \ge 1$ and $d \le 9$. Assume that $d \ge 8$. If $b_5 \ge 19$, then, by Lemma 4, $c_5 \le 19$, a contradiction. From this, $b_5 \le 18$. If $c_4 \ge 20$, then, by Lemma 4, we have $b_4 < 20$, a contradiction. Therefore, $c_4 \le 16$.

If $c_3 \ge 14$, then, by Lemma 4, $b_3 \le 25$. Therefore, $16 - b_4 \ge c_3 - b_3 + 16$ and $c_4 \le b_4 \le b_3 - 14 \le 11$, a contradiction. If $c_3 = 13$, then $b_2 \le 39$ and, by Lemma 4, $b_3 \le 27$. Since b_3 is divided by 4, we have $b_3 \le 24$. Therefore, $16 - b_4 \le 13 - b_3 + 16$ and $c_4 \le b_4 \le b_3 - 13 \le 11$, a contradiction.

The cases $c_3 = 7, 9, 11$ are treated in a similar manner.

Lemma 12. *If* $d > 4$, *then* $\theta_1 > 27$.

Proof. Let $d = 5$. For $\mu \ge 6$, computer enumeration gives $\theta_1 > 38$ and, if $\mu = 4$, then $\theta_1 > 27$.

Let $d = 6$. For $\mu \ge 6$, computer enumeration yields $\theta_1 > 60$ and, if $\mu = 4$, then $\theta_1 > 50$.

Let $d = 7$. For $\mu \ge 6$, computer enumeration produces $\theta_1 > 72$ and, if $\mu = 4$, then $\theta_1 > 59$. The lemma, together with the theorem, is proved.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project no. 12-01-00012 and joint project no. 12-01-91155 with the National Science Fund of China), by the Branch of Mathematics of the Russian Academy of Sciences (project no. 12-T-1-1003), and by the Ural Branch of the Russian Academy of Sciences jointly with the Siberian Branch of the Russian Academy of Sciences (project no. 12-S-1-1018) and with the National Academy of Sciences of Belarus (project no. 12-S-1-1009).

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Translated by I. Ruzanova